<table>
<tr><td rowspan="1"></td><td>P.R.Government College (Autonomous)<br>KAKINADA</td><td rowspan="2" colspan="4">Program&Semester<br>II B.Sc. Major<br>(III Sem)<br>w.e.f 2025-26 admitted batch</td></tr>
<tr><td>Course Code<br>MAT- 304 T</td><td>TITLEOFTHECOURSE<br>SPECIAL FUNCTIONS & Problem Solving Sessions</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:60(Theory)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>Multivariable calculus and Differential Equations</td><td>3</td><td>-</td><td>-</td><td>3</td></tr>
</table>

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

| On Completion of the course, the students will be able to- | |
|---|---|
| CO1 | Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations. |
| CO2 | Find power series solutions of ordinary differential equations |
| CO3 | Solve Hermite equation and write the Hermite Polynomial of order (degree) n, also |
| CO4 | Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials. |
| CO5 | Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction. |

**Course with focus on employability/entrepreneurship /Skill Development modules**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Skill Development | | | Employability | | | Entrepreneurship | |

## UNIT I

**Beta and Gamma functions, Chebyshev polynomials**

Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

Another form of Beta Function, Relation between Beta and Gamma Functions.

Chebyshev polynomials, orthogonal properties of Chebyshev polynomials, recurrence relations, generating functions for Chebyshev polynomials.

**UNIT II:**

**Power series and Power series solutions of ordinary differential equations.**

Introduction, summary of useful results, power series, radius of convergence, theorems on Power series Introduction of power series solutions of ordinary differential equation Ordinary and singular points, regular and irregular singular points, power series solution.

## UNIT III:

**Hermite polynomials**

Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

**UNIT IV:**

**Legendre polynomials**

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials. ,
2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required )to show that $P_n(x)$ is the coefficient of $h^n$ , in the expansion of $(1-2xh+h^2)^{-1/2}$
3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

**UNIT V:**

**Bessel's equation**

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.
2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$. 3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

**Co-Curricular Activities**

Seminar/ Quiz/ Assignments/ Applications of Functions of complex variables to Real life Problem

/Problem Solving Sessions.

**TEXT BOOK:**

Theory of Functions of a Complex variable by Shanti Narayan &Dr. P. K. Mittal, S. Chand &Company Ltd.

**REFERENCE BOOKS:**

1. Theory of Functions of a Complex Variable by A. I. Markushevich, Second Edition, AMS Chelsea Publishing
2. Theory And Applications by M. S. Kasara, Complex Variables, 2nd Edition, Prentice Hall India Learning Private Limited

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SEMESTER-III

| Unit | TOPIC | S.A.Q | E.Q | Marks allotted to the Unit |
|---|---|---|---|---|
| I | Beta and Gamma functions, Chebyshev polynomials. | 2 | 2 | 30 |
| II | Power series and Power series solutions of ordinary differential equations. | 2 | 1 | 20 |
| III | Hermite polynomials | 1 | 1 | 15 |
| IV | Legendre polynomials | 1 | 1 | 15 |
| V | Bessel's equation | 1 | 1 | 15 |
| Total | | 7 | 6 | 95 |

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

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Total Marks = 50 M

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**Pithapur Rajah's Government College (Autonomous), Kakinada**
**II year B.Sc., Degree Examinations - III Semester**
**Mathematics Course VIII: SPECIAL FUNCTIONS**
**Model Paper (w.e.f. 2025-26)**

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**Time: 2Hrs** **Max. Marks: 50**

**SECTION-A**

**Answer any three questions selecting atleast one question from each part**

**Part – A** **3 X 10 = 30**

1. Essay question from unit - I.
2. Essay question from unit - I.
3. Essay question from unit - II.

**Part – B**

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

**SECTION-B**

**Answer any four questions** **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit - I.
9. Short answer question from unit – II.
10. Short answer question from unit - II .
11. Short answer question from unit – III.
12. Short answer question from unit – IV.
13. Short answer question from unit - V

**P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA**

**II B.SC MATHEMATICS MAJOR – III SEMESTER**

# Mathematics Course VIII: SPECIAL FUNCTIONS

**QUESTION BANK**

**Short Answer questions**

## Unit - I

1. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
2. Evaluate $\int_0^1 \frac{dx}{\sqrt{-log_e x}}$
3. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy$ *and hence show that* $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
4. Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$
5. Prove that $(1 - x^2)T^1_n(x) = - n\, x\, T_n(x) + n\, T_{n-1}(x)$
6. Prove that $U_{n+1}(x) - 2x\, U_n(x) + U_{n-1}(x) = 0$
7. Find the first four Chebyshev polynomials.
8. Prove that $T_n(-1) = (-1)^n$ and $U_n(-1) = 0$

## Unit – II

9. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$ .
10. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$
11. Find the radius of the convergence of the series $\sum(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
12. Determine whether x = 0 is an ordinary point or a regular singular point of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right) + 7x(x+1)\frac{dy}{dx} - 3y = 0.$
13. Show that x = 0 and x = -1 are singular points of $x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$ where the first is irregular and the other is regular.
14. Solve by power series method $y' - y = 0$ .

## Unit - III

15. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ *and* $H_{2n+1}(0) = 0.$
16. Find Hermit Polynomials for n=0, 1, 2, 3, 4 .
17. Prove that $H_n'' = 4n(n-1)H_{n-2}$
18. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
19. Prove that $H_n(-x) = (-1)^n H_n(x)$ .
20. Prove that, if $m < n$, $\frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m n!}{(n-m)!}H_{n-m}(x).$

## Unit - IV

21. Prove that $P_n(-x) = (-1)^n P_n(x)$ *and hence deduce that* $P_n(-1) = (-1)^n$

22. Prove that $P_n' = \frac{n(n+1)}{2}$

23. Prove that $(2n+1)P_n = P_{n+1}' - P_{n-1}'$.

24. Prove that $xP_n' - P_{n-1}' = nP_n$.

25. Prove that $(n+1)P_n = P_{n+1}' - xP_n'$.

26. Prove that $(1-x^2)P_n' = n(P_{n-1} - xP_n)$.

27. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

**Unit – V**

28. Prove that, when $n$ is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.

39. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

30. Prove that $J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$

31. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

32. Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\, sin\theta)\, d\theta$

33. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

34. Show that $\int_0^{\infty} e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{(a^2+b^2)}}$ , $a > 0$

**Essay Questions**

**Unit –I**

1. When n is a positive integer, prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5.....(2n-1)}$

2. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

3. Prove that $\int_0^{\pi/2} sin^{2l-1}\,\theta . cos^{2m-1}\theta\, d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

4.State and prove orthogonal properties of Chebyshev polynomials.

5. prove that $T_n(x)$ and $U_n(x)$ are independent solution of Chebyshev's differential equation.

6. Show that $\frac{1}{\sqrt{1-x^2}} U_n(x)$ satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (n^2-1)y = 0$.

**Unit – II**

1. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R .

2. Find the radius of convergence the exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$

3. Determine the interval of convergence of the power series $\sum\{\frac{1}{n}(-1)^{n+1}(x-1)^n\}$

4. Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ in powers of x.

5. Find the solution in series of $\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)+x^2y=0$ about x = 0 .

6. Find the general solution of $y''+(x-3)y'+y=0$ near x = 2 .

**Unit – III**

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. State and Prove Orthogonal Properties of Hermite Polynomials.
4. Prove that $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$.
5. Prove that $H_n'(x)=2nH_{n-1}(x) \quad n\geq 1$ and $H_0'(x)=0$.
6. Prove that $H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}\left(e^{-x^2}\right)$

**Unit – IV**

1. Show that $P_n(x)$ is the coefficient of $h^n$ in the expansion of $(1-2xh+h^2)^{-1/2}$ in

   Ascending powers of h for | x | ≤ 1 and | h | < 1 .

2. Prove that $P_n(x)=\frac{1}{n!2^n}.\frac{d^n}{dx^n}(x^2-1)^n$.
3. Prove that $\int_{-1}^{1}[P_n(x)]^2dx=\frac{2}{2n+1}$ .
4. Prove that $\int_{-1}^{1}P_m(x).P_n(x)dx=0 \;\; if \; m\neq n.$ and 2/(2n+1) if m = n .
5. Prove that $(2n+1)xP_n=(n+1)P_{n+1}+nP_{n-1}$
6. Prove that $\int_{-1}^{1}(x^2-1)P_{n+1}P_n'\,dx=\frac{2n(n+1)}{(2n+1)(2n+3)}$.

**Unit V**

1. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of $z^n$ in the expansion of $e^{\frac{x\left(z-\frac{1}{z}\right)}{2}}$ in ascending and descending powers of $z$.

2. Prove that $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$.

3. Prove that $xJ_n'(x)=-nJ_n(x)+xJ_{n-1}(x)$.

4. Prove that $x^2J''_n(x)=(n^2-n-x^2)J_n(x)+xJ_{n+1}(x)$

5. Prove that $i)\frac{d}{dx}[x^nJ_n(x)]=x^nJ_{n-1}(x) \;\; (ii)\frac{d}{dx}[x^{-n}J_n(x)]=-x^{-n}J_{n+1}(x)$ .

6. Prove that $\sqrt{\frac{\pi x}{2}}J_{3/2}(x)=\frac{1}{x}\sin x\; -\cos x$.

| | | |
|---|---|---|
| | **P.R.Government College (Autonomous)**<br>**KAKINADA** | **Program&Semester**<br>II B.Sc. Major<br>(III Sem)<br>w.e.f 2025-26 admitted batch |
| Course Code<br>MAT- 304 P | **TITLEOFTHECOURSE**<br>**Special Functions &Problem Solving Sessions Practical Course** | |

| | | L | T | P | C |
|---|---|---|---|---|---|
| Teaching | HoursAllocated:30(**Practical**) | L | T | P | C |
| Pre-requisites: | Multivariable calculus and Differential Equations | - | - | 2 | 1 |

## UNIT I

**Beta and Gamma functions, Chebyshev polynomials**

Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

Another form of Beta Function, Relation between Beta and Gamma Functions.

Chebyshev polynomials, orthogonal properties of Chebyshev polynomials, recurrence relations, generating functions for Chebyshev polynomials.

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3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

**UNIT V:**

**Bessel's equation**

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.

2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$. 3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

**TEXT BOOK:**

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**REFERENCE BOOKS:**

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2. Theory And Applications by M. S. Kasara, Complex Variables, 2nd Edition, Prentice Hall India Learning Private Limited

**Semester – III End Practical Examinations**

**Scheme of Valuation for Practical's**

**Time : 2 Hours** **Max.Marks : 50**

- **Record - 10 Marks**
- **Viva voce - 10 Marks**
- **Test - 30 Marks**
- **Answer any 5questions. At least 2 questions from each section. Each question carries 6 marks.**

**BLUE PRINT FOR PRACTICAL PAPER PATTERN**

**COURSE-VIII - SPECIAL FUNCTIONS**

| Unit | TOPIC | E.Q | Marks allotted to the Unit |
|---|---|---|---|
| I | Beta and Gamma functions, Chebyshev polynomials. | 2 | 12 |
| II | Power series and Power series solutions of ordinary differential equations. | 2 | 12 |
| III | Hermite polynomials | 1 | 06 |
| IV | Legendre polynomials | 2 | 12 |
| V | Bessel's equation | 1 | 06 |
| | **Total** | 08 | 48 |

**PITHAPUR RAJAH'S GOVRENMENT COLLEGE (AUTONOMOUS), KAKINADA**
**II year B.Sc., Degree Examinations - III Semester**
**Mathematics Course-VIII: SPECIAL FUNCTIONS**
**(w.e.f. 2024-25 Admitted Batch)**
**Practical Model Paper (w.e.f. 2025-2026)**

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**Time: 2Hrs** **Max. Marks: 50M**

**Answer any 5questions. At least 2 questions from each section.** **5 x 6 = 30 Marks**

**SECTION - A**

1. Unit - I.
2. Unit – I.
3. Unit - II.
4. Unit – II.

**SECTION – B**

5. Unit - III.
6. Unit - IV.
7. Unit – IV.
8. Unit - V.

- **Record - 10 Marks**
- **Viva voce - 10 Marks**